

Autonomous NATCAR Dynamical Control Modeling

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Linearized Dynamics

`Solve[y''[t] == -V^2/L δ[t] - V δ'[t] /.`

`{δ[t] -> Kp y[t] + Kv y'[t], δ'[t] -> Kp y'[t] + Kv y''[t]}, y''[t]]`

`{ {y''[t] -> $\frac{-Kp V^2 y[t] - Kp L V y'[t] - Kv V^2 y''[t]}{L (1 + Kv V)}$ } }`

`MatrixForm[A = {{0, 1}, {-Kp V^2 / (L (1 + Kv V)), (-Kp L V - Kv V^2) / (L (1 + Kv V))}]`

`$\begin{pmatrix} 0 & 1 \\ -\frac{Kp V^2}{L (1 + Kv V)} & \frac{-Kp L V - Kv V^2}{L (1 + Kv V)} \end{pmatrix}$`

Eigenvalue/Stability Analysis

■ Proportional-Derivative Control Eigenvalues:

`Eigenvalues[A] // FullSimplify`

`$\left\{ -\frac{V (Kp L + Kv V + \sqrt{Kp L (-4 + Kp L) - 2 Kp Kv L V + Kv^2 V^2})}{2 (L + Kv L V)}, \right.$`
 `$\left. -\frac{V (Kp L + Kv V - \sqrt{Kp L (-4 + Kp L) - 2 Kp Kv L V + Kv^2 V^2})}{2 (L + Kv L V)} \right\}$`

■ Critical Damping:

`Solve[Kp L (-4 + Kp L) - 2 Kp Kv L V + Kv2 V2 == 0, {Kp, Kd}]`

`Solve::svars` : Equations may not give solutions for all "solve" variables. `More...`

$$\left\{ \left\{ K_p \rightarrow \frac{2 L + K_v L V - 2 \sqrt{L^2 + K_v L^2 V}}{L^2} \right\}, \left\{ K_p \rightarrow \frac{2 L + K_v L V + 2 \sqrt{L^2 + K_v L^2 V}}{L^2} \right\} \right\}$$

■ Proportional Control Eigenvalues:

$$\left\{ -\frac{1}{2(L + K_v L V)} \left(V \left(K_p L + K_v V + \sqrt{K_p L (-4 + K_p L) - 2 K_p K_v L V + K_v^2 V^2} \right) \right), \right. \\ \left. -\frac{1}{2(L + K_v L V)} \left(V \left(K_p L + K_v V - \sqrt{K_p L (-4 + K_p L) - 2 K_p K_v L V + K_v^2 V^2} \right) \right) \right\} /. K_v \rightarrow 0 // FullSimplify$$

$$\left\{ -\frac{(K_p L + \sqrt{K_p L (-4 + K_p L)}) V}{2 L}, \frac{(-K_p L + \sqrt{K_p L (-4 + K_p L)}) V}{2 L} \right\}$$

■ Critical Damping:

`Solve[Kp L (-4 + Kp L) == 0, Kp]`

$$\left\{ \{K_p \rightarrow 0\}, \left\{ K_p \rightarrow \frac{4}{L} \right\} \right\}$$

$$\left\{ -\frac{(K_p L + \sqrt{K_p L (-4 + K_p L)}) V}{2 L}, \frac{(-K_p L + \sqrt{K_p L (-4 + K_p L)}) V}{2 L} \right\} /. K_p \rightarrow 4 / L$$

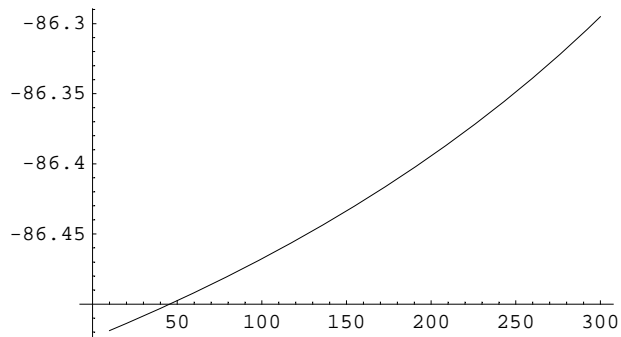
$$\left\{ -\frac{2 V}{L}, -\frac{2 V}{L} \right\}$$

■ Sample Values

$$-\frac{1}{2(L + K_v L V)} \left(V \left(K_p L + K_v V + \sqrt{K_p L (-4 + K_p L) - 2 K_p K_v L V + K_v^2 V^2} \right) \right) /. \\ \{V \rightarrow 20, L \rightarrow 0.230, K_v \rightarrow 0, K_p \rightarrow 3\}$$

`-30. - 65.7069 i`

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Plot[- $\frac{1}{2(L + K_v L V)}$  (V (K_p L + K_v V +  $\sqrt{K_p L (-4 + K_p L) - 2 K_p K_v L V + K_v^2 V^2}$ )) / .  
{V -> 20, L -> 0.230, K_v -> 10}, {K_p, 10, 300}]
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- Graphics -